Reinforcement Learning and Optimal Control

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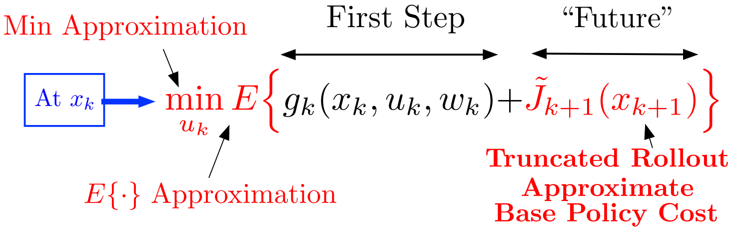
Lecture 5

Outline

The Pure Form of Rollout: For a Suboptimal Base Policy π, Use $\tilde{J}_{k+\ell}(x_{k+\ell}) = J_{k+\ell,\pi}(x_{k+\ell})$



Policy improvement property (where no truncation is used):

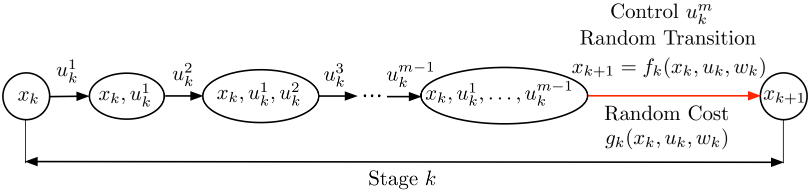
$$J_{k,\tilde{\pi}}(x_k) \le J_{k,\pi}(x_k), \qquad \text{for all } x_k \text{ and } k$$

The key issue in the practical application of rollout: Too much computation

- If the problem is deterministic, the computation is greatly reduced (no Monte Carlo)
- Another computational bottleneck: Large control spaces, e.g., the multiagent case, where controls have many components,

$$u_k = (u_k^1, \ldots, u_k^m)$$

Trading off Control Space Complexity with State Space Complexity



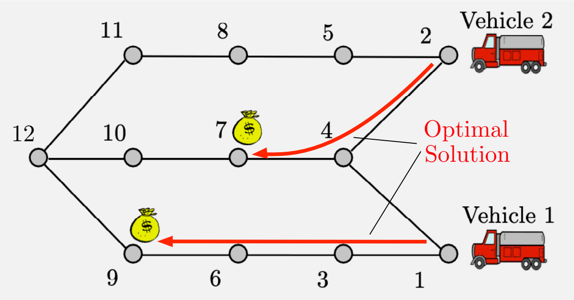
An equivalent reformulation - “Unfolding" the control action

- The control space is simplified at the expense of $m-1$ additional layers of states, and corresponding $m-1$ cost-to-go functions

$$J_k^1(x_k, u_k^1), J_k^2(x_k, u_k^1, u_k^2), \ldots, J_k^{m-1}(x_k, u_k^1, \ldots, u_k^{m-1})$$

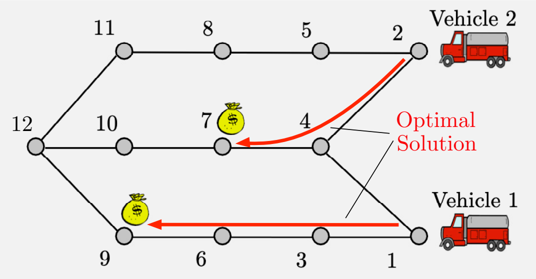
- Multiagent or one-component-at-a-time rollout is just standard rollout for the reformulated problem.
- The increase in size of the state space does not adversely affect rollout.
- The cost improvement property is maintained.
- Complexity reduction: The one-step lookahead branching factor is reduced from n^m to nm, where n is the number of possible choices for each component u_k^i.
- Base policy for u_k^i may depend only on x_k or include dependence on $u_k^{i-1}, u_k^{i-2}, \ldots$.

A Deterministic Example: Multi-Vehicle Routing



- n vehicles move along the arcs of a given graph.
- Some of the nodes of the graph include a task to be performed by the vehicles. Each task will be performed only once.
- Find a route for each vehicle so that all the tasks are collectively performed by the vehicles in minimum time.
- Cost function choice: For each stage there is a cost of one unit for each task that is pending at the end of the stage.
- Total cost: The number of stages that a task is pending, summed over the tasks. What is the state? What is the control? Why is this good multiagent candidate?

Multi-Agent Rollout for Multi-Vehicle Routing

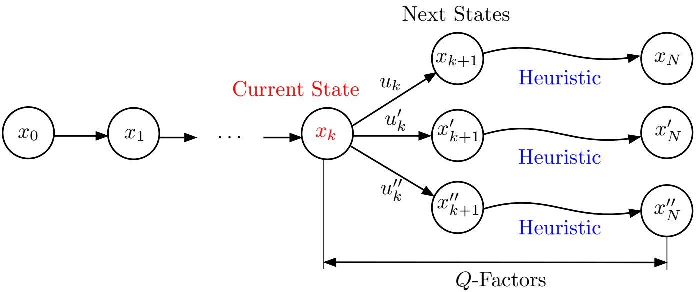


Base heuristic:

- Each vehicle makes a move towards the pending task that is closest (in terms of number of moves needed to reach it).
- The vehicles make their selection independently one-at-a-time in the order $1, \ldots, n$, and take into account the tasks that have already been performed.

- What is the solution produced by the base heuristic?
- What is the solution produced by the one-vehicle-at-a-time rollout?
- Do we get cost improvement? What is the intuition?

Constrained Deterministic Rollout

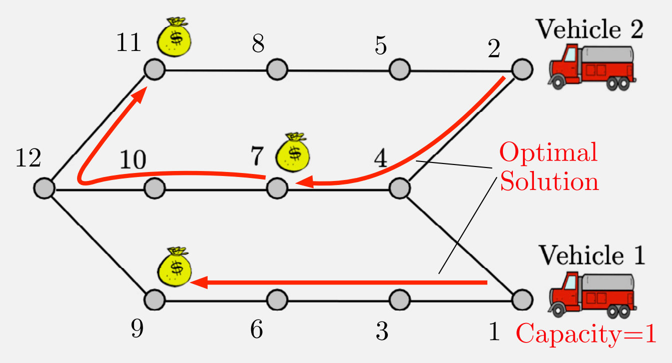


- For every pair (x_k, u_k), $u_k \in U_k(x_k)$, we generate a Q-factor

$$\tilde{Q}_k(x_k, u_k) = g_k(x_k, u_k) + H_{k+1}\big(f_k(x_k, u_k)\big)$$

  using the base heuristic [$H_{k+1}(x_{k+1})$ is the heuristic cost starting from x_{k+1}].
- Select u_k with minimal Q-factor, move to next state x_{k+1} and continue.
- What if there are constraints, i.e., future control choices are constrained by past control choices?
- Base heuristic and rollout should be modified (e.g., avoid controls that compromise feasibility of future controls).

Constrained Example: Multi-Vehicle Routing with Constraints



Examples of constraints

- Vehicle capacity constraints (limit on how many tasks some vehicles can perform).
- Vehicle specializations (some tasks can be performed only by some of the vehicles).
- Time windows (some tasks must be performed within specified time intervals).

How to Deal with Constraints Across Stages in Deterministic Problems

- Consider a deterministic optimal control problem with system $x_{k+1} = f_k(x_k, u_k)$.
- A complete trajectory is a sequence

$$T = (x_0, u_0, x_1, u_1, \dots, u_{N-1}, x_N)$$

- We consider the (very general) problem

$$\min_{T \in C} G(T)$$

where G is a given cost function and C is a given constraint set of trajectories.

We transform to an unconstrained problem in order to apply rollout ideas

- Redefine the state to be the partial trajectory

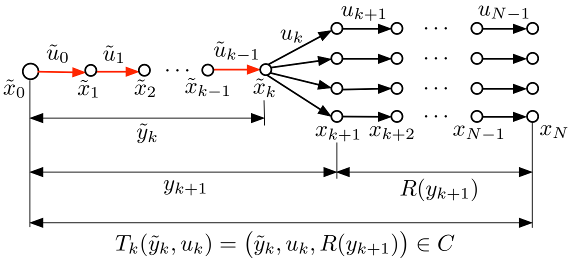
$$y_k = (x_0, u_0, x_1, \dots, u_{k-1}, x_k)$$

- Partial trajectory evolves according to a redefined system equation:

$$y_{k+1} = \big(y_k, u_k, f_k(x_k, u_k)\big)$$

- The problem becomes to minimize $G(y_N)$ subject to the constraint $y_N \in C$.

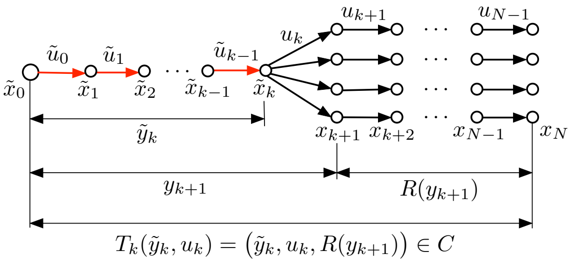
Rollout Algorithm - Partial Trajectory-Dependent Base Heuristic



- Given $\tilde{y}_k = \{\tilde{x}_0, \tilde{y}_0, \tilde{x}_1, \tilde{u}_1, \ldots, \tilde{u}_{k-1}, \tilde{x}_k\}$ consider all controls u_k and corresponding next states x_{k+1}.
- Extend $\tilde{y}_k$ to obtain the partial trajectories $y_{k+1} = (\tilde{y}_k, u_k, x_{k+1})$.
- Run the base heuristic from each y_{k+1} to obtain the partial trajectory $R(y_{k+1})$.
- Join the partial trajectories y_{k+1} and $R(y_{k+1})$ to obtain complete trajectories denoted by $T_k(\tilde{y}_k, u_k) = (\tilde{y}_k, u_k, R(y_{k+1}))$
- Find the set of controls $U_k(\tilde{y}_k)$ for which $T_k(\tilde{y}_k, u_k)$ is feasible, i.e., $T_k(\tilde{y}_k, u_k) \in C$
- Choose the control $\tilde{u}_k \in U_k(\tilde{y}_k)$ according to the minimization

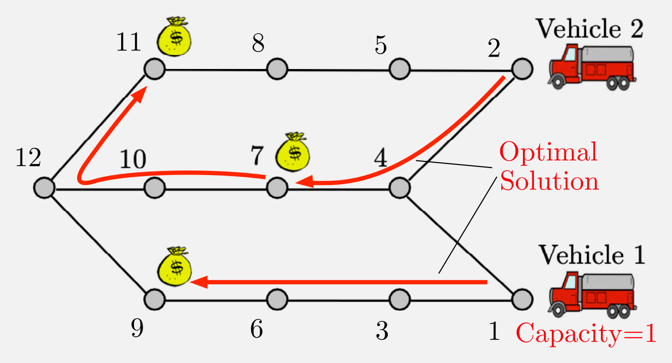
$$\tilde{u}_k \in \arg \min_{u_k \in U_k(\tilde{y}_k)} G(T_k(\tilde{y}_k, u_k))$$

Rollout Algorithm Properties



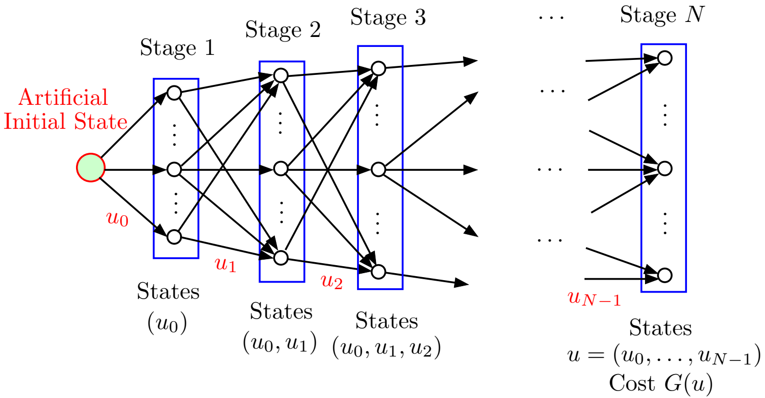
- The notions of sequential consistency and sequential improvement apply. Part of their definition includes that the set of “feasible" controls $U_k(\tilde{y}_k)$ is nonempty for all k; see the notes.
- There is a fortified version (follows the best feasible trajectory). Has the cost improvement property, assuming the base heuristic generates a feasible trajectory starting from the initial condition $\tilde{y}_0 = x_0$
- There is a multiagent version that uses one-control-component-at-a-time minimization.
- Additional variants are possible; see the notes.

Example: Multi-Vehicle Routing with Constraints



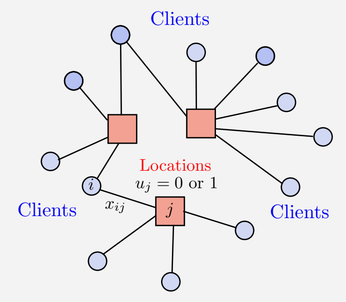
- Base heuristic moves each vehicle (one-at-a-time) to the closest pending task.
- What is the first move of vehicle 1 chosen by base heuristic and by rollout?
- What is the solution found by base heuristic?
- What is the solution found by rollout?

General Discrete Optimization Problem: Minimize $G(u)$ subject to $u \in C$



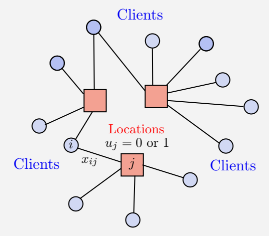
- This is a special case of the constrained deterministic optimal control problem where each state x_k can only take a single value.
- A very broad range of problems, e.g., combinatorial, integer programming, etc.
- Solution by constrained rollout applies. Provides entry point to the use of neural nets in discrete optimization through approximation in policy space.
- Other methods: local/random search, genetic algorithms, etc. Rollout is different.

Facility Location: A Classical Integer Programming Problem



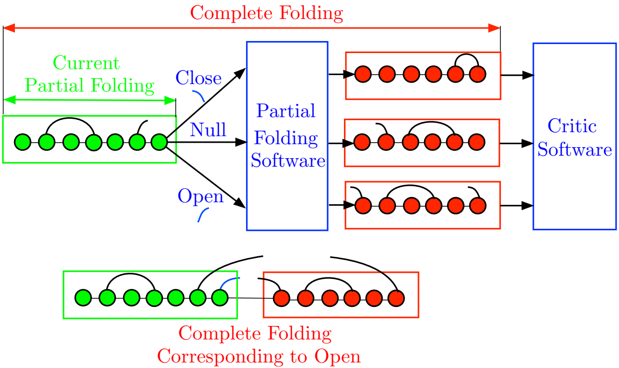
- Place facilities at some locations to serve the needs of M "clients."
- Client $i = 1, \ldots, M$ has a demand d_i for services that may be satisfied at a location $j = 1, \ldots, N$ at a cost a_{ij} per unit.
- A facility placed at location j has capacity c_j and costs b_j. Here $u_j \in \{0, 1\}$, with $u_j = 1$ if a facility is placed at j.
- Problem: Minimize $\sum_{i=1}^{M} \sum_{j=1}^{N} a_{ij} x_{ij} + \sum_{j=1}^{N} b_j u_j$ subject to total demand satisfaction constraints.
- Note: If the placement variables u_j are known, the remaining problem is easily solvable (it is a linear "transportation" problem).

Facility Location Problem: Formulation for Constrained Rollout



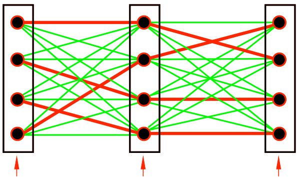
- Consider placements one location at a time.
- Stage = Placement decision at a single location (N stages). State at time k = The placement choices at locations $1, \ldots, k$. Control = Choice between 1 (place) or 0 (not place) for the next facility. What are the constraints? Is this multiagent?
- Base heuristic: Having fixed $u_1, \ldots, u_k$, place a facility in every remaining location.
- Rollout: Having fixed $u_1, \ldots, u_k$, compare two possibilities:
  - Set $u_{k+1} = 1$ (place facility at location $k+1$), set $u_{k+2} = \cdots = u_N = 1$ (as per the base heuristic), and solve the remaining problem.
  - Set $u_{k+1} = 0$ (don't place facility at location $k+1$), set $u_{k+2} = \cdots = u_N = 1$ (as per the base heuristic), and solve the remaining problem.
- Select $u_{k+1} = 1$ or $u_{k+1} = 0$ depending on which yields feasibility and min cost.
- Linear transportation problems can be solved with the auction algorithm.

RNA Folding



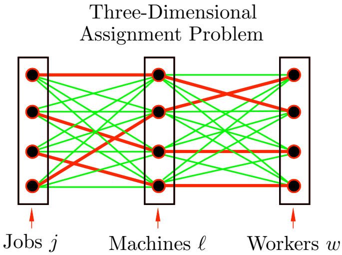
- Given a sequence of nucleotides, “fold" it in an “interesting" way (introduce pairings that result in an “interesting" structure).
- Make a pairing decision at each nucleotide in sequence (open, close, do nothing).
- Base heuristic: Given a partial folding, generates a complete folding (this is the partial folding software).
- Two complete foldings can be compared by the critic software.
- Note: There is no explicit cost function here (it is internal to the critic software).

Three-Dimensional Assignment



- The performance of a job j requires a single machine ℓ and a single worker w.
- There is a given cost $a_{j\ell w}$ corresponding to the triplet (j, ℓ, w).
- Given a set of m jobs, a set of m machines, and a set of m workers, we want to find a collection of m job-machine-worker triplets that has minimum total cost.
- A favorable case is when the cost has separable form $a_{j\ell w} = \beta_{j\ell} + \gamma_{\ell w}$
- Enforced separation heuristic:
  - First solve an artificial 2-dimensional machines-to-workers assignment problem with costs $\gamma_{\ell w}$ derived from $a_{j\ell w}$, e.g., $\gamma_{\ell w} = \min_j a_{j\ell w}$ (the “optimistic" assignment costs).
  - Next solve the 2-dimensional jobs-to-machines assignment problem with costs $\beta_{j\ell}$ specified by the machines-to-workers assignment and $a_{j\ell w}$.
- 2-D assignment problems are easy (using the auction algorithm; see the notes).

Three-Dimensional Assignment: Use of Rollout



- View as a constrained multistage problem.
- Two stages; control at stage 1 = the jobs-to-machines assignment; control at stage 2 = the machines-to-workers assignment
- We view stage 1 assignment as “multiagent": Assign one job at a time.
- We view stage 2 assignment as “single-agent": Assign all machines at once optimally (given the stage 1 assignment).
- Base heuristic: Having fixed some stage 1 assignments, use enforced separation heuristic for the remaining stage 1 assignments, and the stage 2 assignment.
- Rollout: Fix each job assignment one-at-a-time using the base heuristic to compare all machine options.

About the Next Two Lectures

We will cover:

- Parametric approximation architectures.
- Neural networks and how we use them.
- Approximation in value space and policy space using neural nets.

PLEASE READ AS MUCH OF CHAPTER 3 AS YOU CAN

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